COSMOS description

CWE Market Coupling algorithm

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Related documents

Attachments

1 Summary

The CWE project parties have selected COSMOS as the algorithm to calculate daily market coupling results. COSMOS is a branch-and-bound algorithm designed, in collaboration with N-SIDE, to solve the problem of coupling spot markets including block orders. It naturally treats all technical and product requirements set by the CWE project, including step and interpolated orders, flow-based network under PTDF representation, ATC links and DC cables (possible with ramping, tariffs and losses), profiles block orders, flexible blocks orders and linked block orders.

COSMOS outputs net export positions and prices on each market and each hour, the set of executed orders, and the congestion prices on each tight network element. These outputs satisfy all requirements of a feasible solution, including congestion price properties and the absence of Paradoxically Accepted Blocks.

This document only describes the features that are currently in use in the CWE context, though COSMOS already integrates many additional features such as those to be expected in a context of product and geographic extensions.

As such, COSMOS algorithm is also used for local and coupling auctions such as:
- EPEX SPOT for CH;
- HUPX, OTE and OKTE for CZ-HU-SK coupling.

In those market areas, the COSMOS market constraints are defined in local market rules.

2 General principles of market coupling

2.1. General principle of market coupling

Market coupling is both a mechanism for matching orders on power exchanges (PXs) and an implicit cross-border capacity allocation mechanism. Market coupling optimizes the economic efficiency of the coupled markets: all profitable deals resulting from the matching of bids and offers in the coupled markets of the PXs are executed; matching results are however subject to capacity constraints calculated by Transmission System Operators (TSOs) which may limit the flows between the coupled markets.

Market prices and schedules of the connected markets are simultaneously determined with the use of the available capacity defined by the TSOs. The transmission capacity is thereby implicitly auctioned and the implicit cost of the transmission capacity is settled by the price differences between the markets. In particular, if no transmission capacity constraint is active, then there is no price difference between the markets and the implicit cost of the transmission capacity is null.

2.2. ATC market coupling

Under ATC, Market coupling relies on the principle that the markets with the lowest prices export electricity to the markets with the highest prices. Between two markets, two situations are possible: either the ATC is large enough and the prices of both markets are equalized (price convergence), or the ATC is not sufficient and the prices cannot be equalized. These two cases are described in the following examples.

Suppose that, initially, the price of market A is lower than the price of market B. Market A will therefore export to market B, the price of market A will increase whereas the price of market B decreases. If the ATC from market A to market B is sufficiently large, a common price in the market may be reached (PA* = PB*). This first case is illustrated in Figure 1.

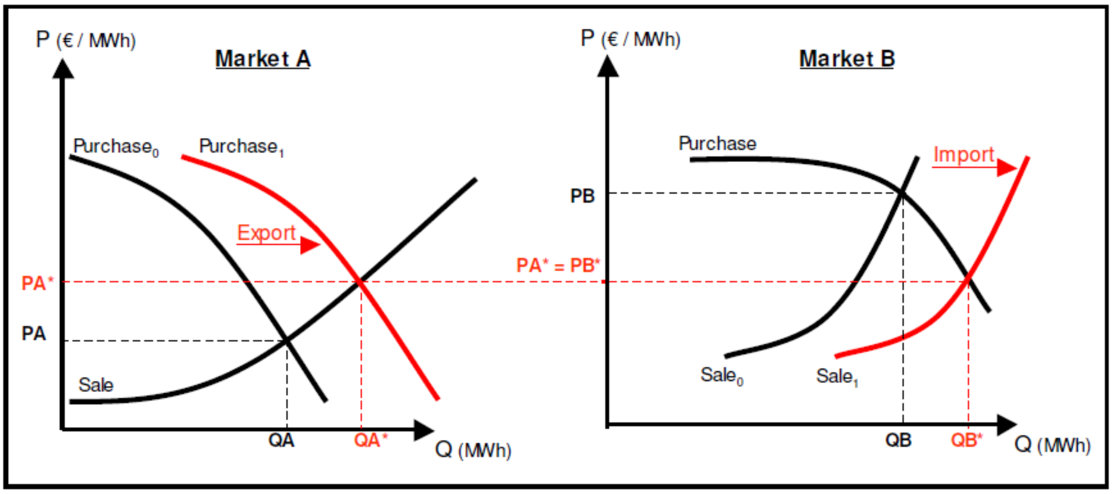


Figure 1: Market coupling of two markets with no congestion

The other case, illustrated in Figure 2, happens when the ATC is not sufficient to ensure price harmonization between the two markets. The amount of electricity exchanged between the two countries is then equal to the ATC and the prices PA* and PB* are given by the intersection of the purchase and sale curves. Exported electricity is bought in the export area at a price of PA* and is sold in the import area at a price of PB*. The difference between the two prices multiplied by the exchanged volume – i.e. the ATC – is called "congestion revenue", and is collected and used pursuant to article 6.6 of the Regulation (EC) N° 1228/2003 of the European Parliament and of the Council of 26 June 2003 on condition for access to the network for cross-border exchanges in electricity.

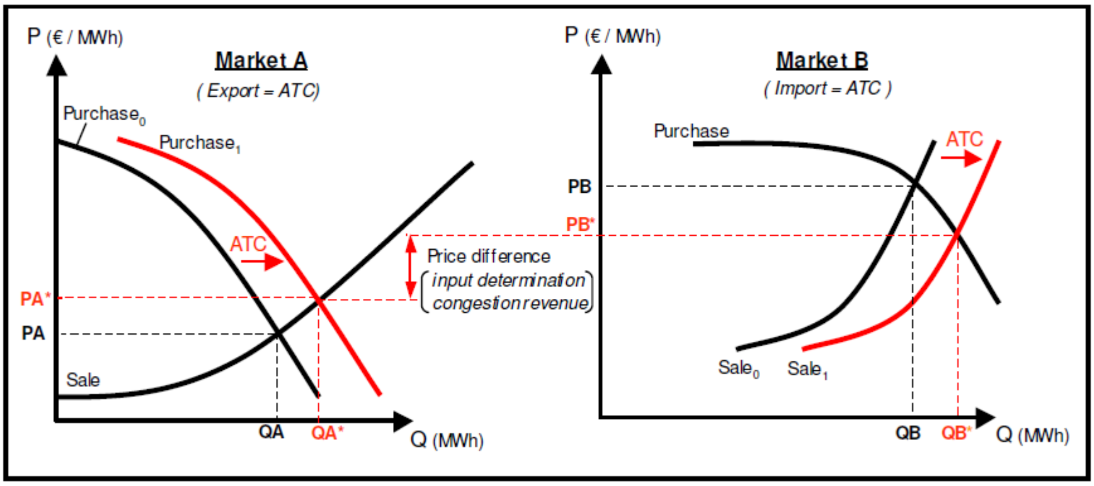


Figure 2: Market coupling of two markets with congestion

3 COSMOS in a nutshell

This section describes the model and the algorithm that have been chosen to solve the problem associated with the coupling of the day-ahead power markets in the CWE region.

Market participants submit orders on their respective power exchange. The goal is to decide which orders to execute and which to reject and publish prices such that:

- The social welfare[1] generated by the executed orders is maximal.
- Orders and prices are coherent.
- The power flows induced by the executed orders, resulting in the net positions do not exceed the capacity of the relevant network elements.

COSMOS has been initially developed by BELPEX, in collaboration with N-SIDE, a company specialized in optimization solutions based on operations research & modeling. The purpose of this algorithm is to deal with the CWE coupling problem in way that allows considering more general aspects of market coupling such as constraints that would arise if coupling with neighboring markets. COSMOS is currently co-owned by APX-ENDEX, BELPEX and EPEX SPOT.

In summary, the COSMOS algorithm:

- naturally treats standard and “new” order types with all their requirements,
- naturally handles both Available Transmission Capacity (ATC) and Flow-Based (FB) network representations as well as possible alternative models and HVDC cable features,
- implements specific curtailment rules for those cases where price boundaries are not harmonized
- is not limited by the number of markets, orders or network constraints,
- finds quickly (within seconds) a very good solution in all cases (even with problems with 350000 hourly orders and 1800 block orders in more than 10 markets),
- continues improving this initial solution until the time limit (e.g. 10 min) is reached,
- generating several feasible solutions during the course of its execution,
- unless it can show that the mathematically optimal solution has been found (which is most often the case), in which case it stops before the time limit.

In the two following sections, we detail which products and network models can be handled by COSMOS. Section 6 gives a high-level description of how COSMOS works, and section 7 provides additional information related to the functionalities and behaviors of the algorithm.

4 Market constraints

Market constraints[2] are those applying to the orders submitted to the exchanges. The list presented hereunder proposes a set of all products available in at least one CWE Exchange.

4.1. Hourly orders

Depending on markets needs and on already existing systems, hourly orders can be either stepwise (BELPEX, APX-ENDEX,) or linearly interpolated (EPEX SPOT).

The fixing of hourly orders satisfies the following constraints:

[1] Social welfare is defined as: consumer surplus + producer surplus + congestion revenue across the region. It is the objective function of COSMOS (see “objective” in technical appendix 1).

[2] See also “Market constraints” section in appendix 1 for a mathematical formulation of these constraints.

- An Hourly Offer is rejected when the Market Clearing Price is lower than the offer (lowest) price limit.
- An Hourly Bid is rejected when the Market Clearing Price is higher than the bid (highest) price limit.
- An Hourly Offer is executed when the Market Clearing Price is higher than the offer (highest) price limit.
- An Hourly Bid is executed when the Market Clearing Price is lower than the bid (lowest) price limit.
- An Hourly Order may be partially executed if and only if the Market Price is equal to the price limit of that order / is between the two price limits of that order.
- An Hourly Order is not executed for a quantity in excess of the volume limit specified in the Order.

4.2. Profile block orders

Compared to block orders that were available prior to the CWE market coupling go-live, block orders in COSMOS can represent profiles, i.e. are defined by distinct volume limits at each hour.

Block orders are neither partially nor paradoxically executed. Therefore, all orders can only be either executed fully, or rejected fully. Because of this constraint – called the "fill or kill constraint" - some block orders can be rejected even if they are in the money[3], in which case they are called Paradoxically Rejected Blocks (PRB). On the contrary, no block orders can be executed paradoxically (i.e. executed even if out of the money).

The fixing of block orders satisfies the following constraints:

- A Block Offer is not executed when the average of the rounded Market Clearing Prices over the relevant hours and weighted by the corresponding volume limits is lower than the price limit of this order.
- A Block Bid is not executed when the average of the rounded Market Clearing Prices over the relevant hours and weighted by the corresponding volume limits is higher than the price limit of this order.
- A Block Order can only be executed at all hours simultaneously, for a quantity equal to the hourly volume limits specified in the order.

5 Network Constraints

COSMOS is able to tackle the network constraints associated with several network configurations[4] (ATC-Based and Flow-Based – as well as with HVDC cables and ramping constraints in case of further extensions).

5.1. ATC-Based constraints

With an ATC-Based representation of the network, the cross border bilateral exchanges are only limited by the ATCs as provided for each hour and each interconnection in both directions. The algorithm will thus compute the cross border bilateral exchanges that are optimal in terms of overall social welfare.

The fixing with ATC-Based constraints satisfies the following constraints:

- Cross-border bilateral exchanges are smaller than or equal to the relevant ATC value.
- Whenever the cross-border exchange is strictly smaller than the relevant ATC value, then the clearing prices on both sides of the border are equal.

[3] A supply (respectively demand) order is said to be in the money if the submission price of the order is below (resp. above) the average market price.
[4] See also "Network constraints" section in appendix 1 for a mathematical formulation of these constraints.

- Whenever there is a price difference between two areas, the ATC between these, if any, is congested in the direction of the high price area.

5.2. Flow-Based constraints (used for FB parallel runs)

Flow-based network representations are set to model more precisely physical electricity laws.

In a flow-based representation of the network, the flows on a set of given critical network elements are equal to the product of a PTDF (Power Transfer Distribution Factor) matrix with the vector of the areas' net positions. These (unidirectional) flows are limited by the corresponding transmission capacities provided for each hour. Such PTDF constraints allow representing explicitly all critical elements and security constraints, but would also support more simplified network models.

The fixing with Flow-Based constraints satisfies the following constraints:

- For each flow-based constraint, the sum of the area's net positions of all markets weighted by the PTDF value is smaller than or equal to the corresponding transmission capacity.
- The sum of the areas' net position is equal to zero
- For each flow-based constraint, whenever the sum of the area's net positions of all markets weighted by the PTDF value is strictly smaller than the corresponding transmission capacity, then the congestion price of this constraint is null.
- The price difference between two areas is equal to the sum of the congestion prices of all capacity constraints weighted by the difference of the corresponding PTDF values.

6 Functioning of COSMOS

In this section we describe how COSMOS selects orders to be executed or rejected, under the Market and Networks Constraints. The objective of COSMOS is to maximize the social welfare, i.e. the total market value of the day-ahead auction.

The main difficulty in determining which orders to execute or reject comes from the fact that block orders must satisfy the "fill or kill" property.

Without those block orders, the problem is much simpler to solve. Indeed, the problem can then naturally be modeled as a Quadratic Program (QP)[5], which can be routinely solved by off-the-shelf commercial solvers[6]. The use of a commercial solver to directly solve this Quadratic Program would then be the most efficient solution.

The presence of block orders in the order book however makes the problem substantially more difficult. The problem with block orders can be formulated as a Mixed Integer Quadratic Program (MIQP) allowing modeling the fill or kill condition of block orders. The state-of-the-art method used to solve MIQP is called branch-and-bound[7]. COSMOS has been designed as a dedicated branch-and-bound algorithm for solving the CWE Market Coupling problem (the mathematical model of this problem is proposed in appendix 1).

6.1. Algorithm

COSMOS proceeds step by step.
At the first step, COSMOS solves a market coupling QP without fill or kill constraints, hence allowing all block orders to be partially executed. By chance, the solution of this problem might satisfy the fill or kill condition for all block orders and is therefore a feasible solution

[5] A Quadratic Program (QP) is an optimization problem where an objective (function) of the second order is to be optimized under linear constraints.
[6] COSMOS uses the CPLEX solver.
[7] For a more extensive discussion on the branch and bound technique, see for instance: Integer Programming, Wolsey, 1998

of the CWE market coupling problem. In this case, the solution that has been found is the optimal solution.

Otherwise, COSMOS gradually forces the partially executed block orders to be either fully rejected or fully executed in subsequent steps, in order to obtain a solution to the CWE market coupling problem which respects all fill or kill constraints.

At a given step, two situations can occur:

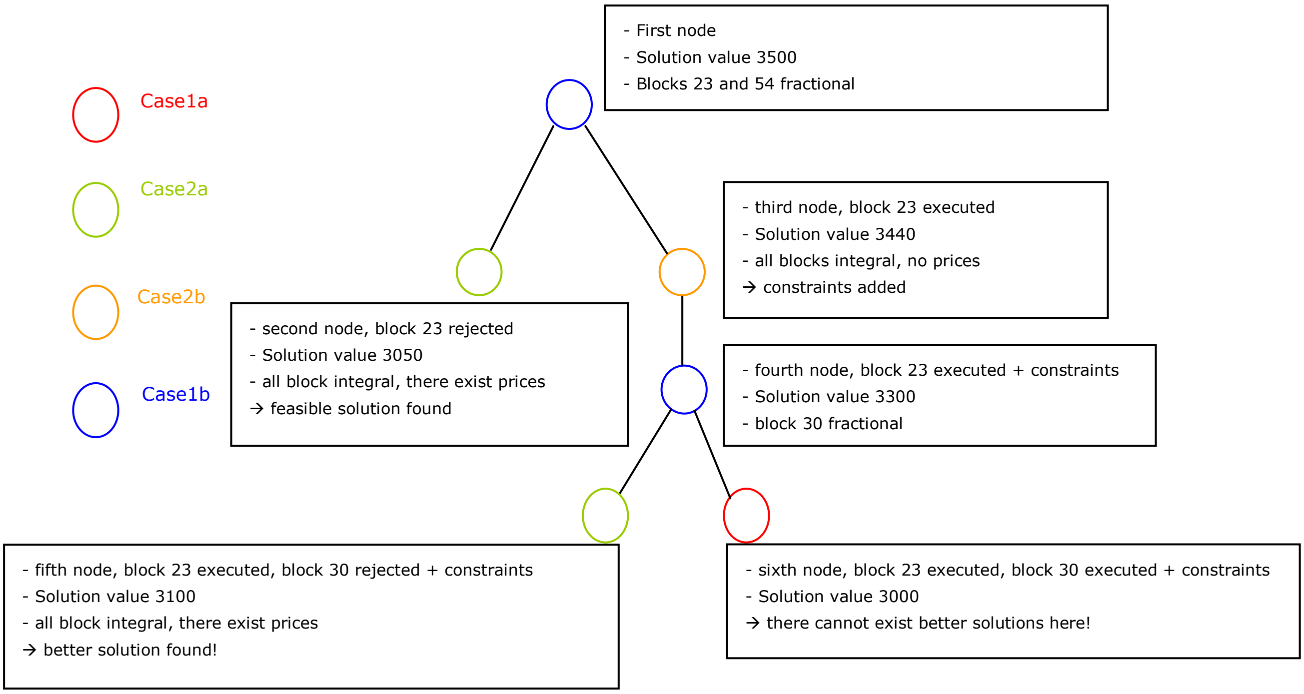
1. COSMOS has produced a solution in which some block orders are either fully executed or rejected and some block orders are partially executed. This solution has been computed by solving the initial QP, but in which some block orders have been forced to be executed or rejected (as the result of some previous steps). Since it contains partially executed orders, it is called a partial solution. The property of this solution is that its objective value is an *upper bound* of the welfare of any solution that could be produced by extending this partial solution into a feasible solution by adding further constraints. Two sub-cases can occur:
   - Sub-case 1a: If the upper bound associated to this partial solution is smaller than the welfare of the best feasible solution found so far, COSMOS will discard this partial solution and won't consider it anymore.
   - Sub-case 1b: Otherwise, COSMOS will select a block order partially executed and create two new steps to be analyzed: in the first of these new steps, the selected block is forced to be executed, and in the second one it is forced to be rejected.

2. COSMOS has produced a solution in which all block orders are either fully executed or fully rejected (even those that were not forced to). In this case, COSMOS must still check whether there exist prices that are compatible with this solution and the constraints (which is done by verifying that all market and network constraints are satisfied). Two cases can occur:
   - Sub-case 2a: If such prices exist, COSMOS has found a feasible solution. If this solution is better than the best one found so far[8], it is marked as such.
   - Sub-case 2b: If no such prices exist, then a new step is created with a transformed problem containing additional constraints to exclude this non feasible solution.

During the course of its execution, COSMOS might sometimes increase the number of steps that it has yet to consider (e.g. sub-cases b) or reduce it (sub-cases a). When there remains none, this means that COSMOS has finished and has found the best possible solution. Possibly, COSMOS will reach the time limit although there remain some partial solutions that were not analyzed. In this case, COSMOS will output the best solution found so far without being able to prove whether it is the very best possible one.

[8] Or if it is the first feasible solution found.

Here is a small example of the execution of COSMOS:



6.2. Precision and rounding

COSMOS provides exact results which satisfy all constraints with a target tolerance of 10^{-5} (and in any case 10^{-3}). Those exact prices and volumes (net positions) are rounded by applying the commercial rounding (round-half-up) convention before being published. The size of the tick varies depending on the data considered. For instance:

- Prices at APX-ENDEX BELPEXSPOT are rounded with two digits (e.g. 0.01 €/MWh)
- Prices at EPEX SPOT are rounded with three digits on the FR market (e.g. 0.001 €/MWh) and with two digits on the DE/AU market
- Net positions in BE, DE and NL are rounded with 1 digit (e.g. 0.1 MWh)
- Net positions in FR are rounded with no digit (e.g. 1 MWh)

NB: rounding the results imposes to accept some tolerances on constraints. Typically, this tolerance is equal to the sum the precision ticks of all rounded values divided by two. For instance, the sum of the net positions of all bidding areas must be zero, with a tolerance of 0.65 MWh (the sum of the net position ticks of all markets divided by two) for the CWE MC.

6.3. Price boundaries

Published prices must be within predefined boundaries. It is intended that all price boundaries will be harmonized so that prices are in the [-3000 €/MWh,+3000 €/MWh] interval, though the algorithm is designed to also support different price boundaries

6.3.1. Price boundaries and network constraints

Generally speaking, different price boundaries can be implemented in COSMOS, but not together with the network price properties as commonly defined. In particular, flow-based models in general hinder the possibility to impose boundaries on prices at all in some particular cases.

In order to accommodate technical price boundaries and to compute coherent prices (in the sense that they respect market and network constraints), COSMOS guarantees on the one hand that market and network constraints are satisfied with respect to unrounded prices. On the other hand, COSMOS also ensures that market constraints are satisfied using rounded and within bound prices. Hence some network constraints are not checked against rounded and within bound prices, but only against unrounded and possibly out of bounds prices. This allows computing coherent prices while respecting the local price boundaries and is currently only relevant in flow-based simulations.

See technical appendix 2 for more information.

6.3.2. Extreme prices and curtailment

Generally speaking, Cosmos is designed to avoid curtailment situations – i.e. situations when price taking orders are not fully satisfied. More precisely, COSMOS enforces local matching of price taking hourly orders with hourly orders in the opposite direction and in the same market as counterpart. Hence, whenever curtailment of price taking orders can be avoided locally on an hourly basis – i.e. the curves cross each other - then it is also avoided in the final results. In case the local matching does not allow to fulfill all the price taking orders, then this curtailment is equally apportioned between all markets (subject to network constraints).

See technical appendix 3 for more information.

6.4. Optimality and quality of the solution

During the course of its execution, COSMOS will typically generate several feasible solutions. The best one in terms of welfare is selected among these solutions at termination of the algorithm.
By optimizing welfare, COSMOS also avoids – whenever possible - generating solutions with paradoxically rejected orders (PRBs), and especially the ones with large volume and/or largely in the money.

COSMOS algorithm selects the solutions with the largest welfare, but discards during its computation the solutions with paradoxically rejected blocks that are very deep in the money. This is implemented to guarantee fairness, as this could only happen with blocks of small volume.

6.5. Time control

COSMOS is tuned to provide very quickly a first feasible solution. It can be shown that the upper bound in terms of computing time to obtain a first feasible solution is linear in terms of number of block orders. In practical cases, the first feasible solution has been found within less than 30 seconds on all our CWE instances.

Due to the combinatorial complexity of the problem, this is obviously not true for the computing time to obtain the optimal solutions. Nevertheless, most of the instances were solved at optimality in less than 10 minutes (which is the maximal time allowed in the CWE coupling process), the remaining showing quite small distances to optimality after this time limit.

6.6. Transparency

Generally speaking, COSMOS is based on sound and robust concepts and has a good degree of transparency. In particular, COSMOS is perfectly clear and transparent as to what are feasibility and optimality. More precisely, COSMOS will typically consider all feasible solutions and choose the best one according to the agreed criterion (welfare-maximization).

Also, COSMOS optimizes the total welfare, so that the chosen results are well explainable to the market participants: published solutions are the ones for which the market value is the largest. In addition, in order to avoid undesirable solutions, COSMOS will not output solutions in which blocks that are unduly deep in the money are rejected paradoxically.

7 Further geographic and product extensions

During the design and implementation of COSMOS, great care has been taken to ensure that the additional requirements aiming at supporting potential extensions in the product range or the geographical scope of the coupling (or possibly both of them) are also met. In particular, all the foreseeable requirements necessary to facilitate the mid-term extendibility of the COSMOS solution (linked and flexible orders of NPS, ramping constraint of NorNed, charges and losses of BritNed and IFA) are already supported by COSMOS.

Furthermore, COSMOS uses a very general method for solving the market coupling problems with “fill or kill” constraints. The ability of the algorithm to handle new products or new requirements is thus excellent as long as the constraints remain of the same type (linear constraints, with possible fill or kill conditions).

8 APPENDIX 1: Mathematical formulation

In this appendix, we describe the mathematical formulation of the Market Coupling Problem that is solved by COSMOS. As mentioned earlier, it is a Mixed-Integer Quadratic Program (MIQP).

8.1. Sets

Let us first introduce some sets, and their indices:
- Bidding areas : **m**
- Hours: **h**
- Hourly orders: **o**
- Profile block orders: **b**
- Unidirectional ATC lines: **l**

8.2. Data

Here are the input data of the Coupling problem:
- $\mathbf{q_o}$ is the quantity of hourly order o; it is considered positive for supply orders and negative for demand orders
- $\mathbf{p^0_o}$ is the price at which an hourly order starts to be accepted
- $\mathbf{p^1_o}$ is the price at which an hourly order is fully accepted (in the case of step orders, $p^0_o = p^1_o$; in the case of interpolated supply orders, $p^0_o \le p^1_o$; in the case of interpolated demand order $p^0_o \ge p^1_o$)
- $\mathbf{q_{b,h}}$ is the quantity of profile block b on period h; it is considered positive for supply orders and negative for demand orders
- **bidding area (b)** is the bidding area in which profiled block order b originates
- **hours(b)** is the set of hours on which profile block order b spans
- **bidding area(o)** is the bidding area in which hourly order o b originates
- **hour(o)** is the hour on which hourly order o spans
- **from(l)** is the bidding area from which line l is originating
- **to(l)** is the bidding area to which line l is leading
- $\mathbf{capacity_{l,h}}$ is the capacity on the ATC line l on period h

8.3. Variables

- $0 \le \mathbf{ACCEPT_o} \le 1$ Acceptance of the hourly order
- $\mathbf{ACCEPT_b} \in \{0,1\}$ Acceptance of the block order
- $0 \le \mathbf{FLOW_{l,h}}$ Flow on the line l at period h
- $\mathbf{MCP_{m,h}}$ Market clearing price at market m and period h
- $0 \le \mathbf{ATC\_PRICE_{l,h}}$ Congestion price of the capacity constraint of line l at period h

8.4. Market Constraints

These are the constraints linking the prices to the orders selection.

- An hourly order o may be accepted only if it is at- or in-the-money:

$$ACCEPT_o > 0 \Rightarrow q_o \cdot \left(MCP_{biddingarea(o),hour(o)} - p_o^0\right) \ge 0$$

- An hourly order o must be refused if it is out-of-the-money:

$$q_o \cdot \left(p_o^0 - MCP_{biddingarea(o),hour(o)}\right) > 0 \Rightarrow ACCEPT_o = 0$$

- An hourly order o may be partially rejected only if it is at-the-money:
  $$0 < ACCEPT_o < 1 \Rightarrow MCP_{bidding area(o),hour(o)} = p_o^0 + \left(p_o^1 - p_o^0\right) \cdot ACCEPT_o$$
- An hourly order o must be fully accepted if it is in-the-money:
  $$q_o \cdot \left(p_o^1 - MCP_{bidding area(o),hour(o)}\right) < 0 \Rightarrow ACCEPT_o = 1$$
- An accepted block b must be in-the-money:
  $$ACCEPT_b = 1 \Rightarrow \sum_{h \in hours(b)} q_{b,h} \cdot \left(MCP_{bidding\ area(b),h} - p_b\right) \geq 0$$

8.5. Network Constraints

These are the constraints on the physical exchanges and the prices arising from the ATC network model of the CWE region.

- The bidding area m must be in balance on hour h, meaning that the sell and the import volumes must match the purchase and the export volumes:
  $$\sum_{\substack{o, bidding\ area(o)=m,\\ hour(o)=h}} ACCEPT_o \cdot q_o + \sum_{\substack{b, bidding\ area(b)=m,\\ h \in hours(b)}} q_{b,h} ACCEPT_b = \sum_{l,\ from(l)=m} FLOW_{l,h} - \sum_{l, to(l)=m} FLOW_{l,h}$$

- The maximum flow on an ATC line l on hour h must not exceed the available capacity.
  $FLOW_{l,h} \leq Capacity_{l,h}$
- The congestion price of an ATC line l on hour h can be positive only if the line is congested.
  $ATC\_PRICE_{l,h} > 0 \Rightarrow FLOW_{l,h} = Capacity_{l,h}$

- The (positive) congestion prices of the capacity constraints must be equal to the price difference between the destination and the source bidding areas of the ATC line l on hour h:
- $ATC\_PRICE_{l,h} = MCP_{to(l),h} - MCP_{from(l),h}$

8.6. Objective

The objective of the Market Coupling Problem is to maximize the total welfare:

$$\max \sum_o - q_o ACCEPT_o \left(\frac{p_o^0 + p_o^1}{2} + \frac{p_o^0 - p_o^1}{2} \left(1 - ACCEPT_o\right) \right) - \sum_b p_b ACCEPT_b \sum_{h \in hours(b)} q_{b,h}$$

Here the welfare is defined as the difference between the cumulative amount that the buyers are ready to pay and the cumulative amount that the sellers want to be paid (quantities are signed) over all markets and hours. This difference corresponds to the sum of the surplus of the producers and the consumers, plus the congestion revenue.
Note that the objective is quadratic in the $ACCEPT_o$ variables (for interpolated orders).

8.7. Summary

COSMOS is an algorithm that generates several (i.e. virtually all) solutions which satisfy the constraints listed under "Market Constraints", and "Network Constraints" and chooses the best one according to the criteria formulated under "Objective".

9 APPENDIX 2: Price indeterminacy rules

There are cases where multiple solutions of the Market Coupling Problem have identical block selections and identical welfares but different clearing prices. In the most frequent cases, this arises when offer and demand curves overlap on a vertical segment. These cases are called price indeterminacies. The aim of this appendix is to describe the rules implemented in COSMOS to choose amongst these clearing prices.

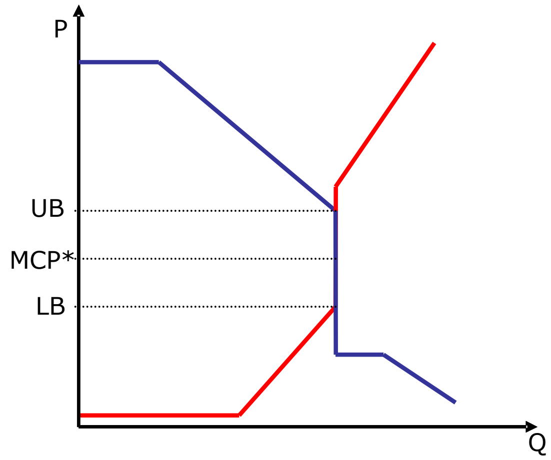


Figure 3: Simple case of price indeterminacy

In a nutshell, COSMOS will pick the mid-point of the intersection, that is MCP* in the figure above. However, in a realistic problem with several markets coupled over several hours, this solution might be non-feasible. This would be the case for example if MCP* invalidates the selection of block (i.e. the price acceptance of block orders are not compatible with MCP*). In such a case, COSMOS will pick up the price which is closed to the mid point and which respects all other price constraints.

This section explains in details how COSMOS selects prices in indeterminate cases[9].

9.1. Notations

Let's note:
$\mathbf{LB_{m,h}}$: the lowest possible price in the market m on hour h for a defined net position Q*.
$\mathbf{UB_{m,h}}$: the highest possible price in the market m on hour h for a defined net position Q*.
$\mathbf{MCP_{m,h}}$: the (unrounded) Market Clearing Price of the market m on hour h.

The mid point of the intersection defined by $[LB_{m,h}, UB_{m,h}]$ is equal to $(UB_{m,h}+LB_{m,h})/2$.

9.2. Mid point rule formulation for price determination

A potential feasible solution produced by the algorithm is definitively a feasible solution once unique rounded prices satisfying all constraints can be defined for this solution. In case of non-unique price solution, for a given hour h, the mid-point rule is defined as the minimal square distance between $MCP_{m,h}$ and the distance to its "mid point". In case of several indeterminacies, the sum of these distances is considered. The mid-point price expression can be formulated with the following function

[9] Note that these rules are independent of the ATC or FB network model

$$\min \sum_{m,h} \left(MCP_{m,h} - \frac{UB_{m,h} + LB_{m,h}}{2} \right)^2$$

Subject to the following constraints:

- All market and network constraints
- B_ACCEPT_o = 1 if the block order o is executed in the feasible solution
  = 0 otherwise

9.3. Prices to be published

These prices must be rounded and bounded before being published. This is done as follows

$$Published\_price_{m,h} = \min(\max(round(MCP_{m,h}, precision_m), P\min_m), P\max_m)$$

With:

- $Published\_Price_{m,h}$: the price published by the Exchange in market m on hour h
- $Precision_m$: the precision of prices in market m
- $Pmin_m$: the lower price boundary of market m
- $Pmax_m$: the upper price boundary of market m

Note that market constraints are checked against these prices as well before being published

9.4. Summary

COSMOS lifts potential price indeterminacies by selecting the prices at the middle of the feasible interval, taking into account the price constraints arising from block selection and network configuration. Then Cosmos rounds and enforces bounds on these prices before publication.

10 APPENDIX 3: Volume indeterminacies and curtailment rules

There are cases where multiple solutions of the Market Coupling Problem have identical prices, identical block selections and identical welfares but different volumes sold or bought on some markets. The cases are called volume indeterminacies. The aim of this appendix is to describe the rules implemented in COSMOS to choose amongst these solutions.

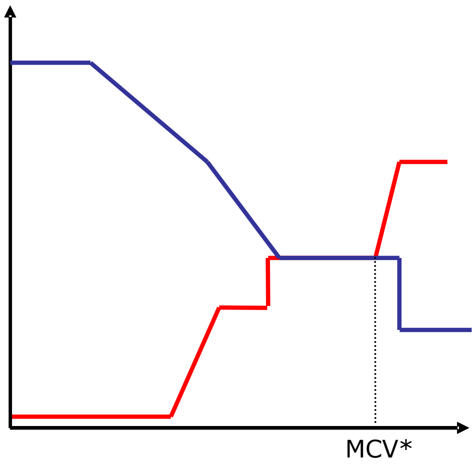


Figure 4: Simple case of volume indeterminacy

In a nutshell, COSMOS will choose the solutions for which the traded volumes are maximal, that is MCV* in the figure above.

In addition, these volume indeterminacy rules encompass curtailment rules which are rules for special volume indeterminacy situations, namely at minimum/maximum prices. Indeed, a secondary objective of COSMOS is to avoid that price taking orders - i.e. orders at extreme prices - are not fully executed, this situation being called *curtailment*[10]*.*

10.1. Definitions and objective

Volume indeterminacy occurs when, for

- a given set of market clearing prices (and congestion prices) satisfying the constraints of the COSMOS model
- a given feasible block selection

There exists many solutions for the hourly orders such that

- they are compatible with the market clearing prices (i.e. out-of-the-money orders are entirely rejected and in-the-money orders are entirely executed)
- they are compatible with congestion prices (i.e., elements with non-zero congestion prices are congested).

In this case, all these solutions have an identical welfare. So welfare cannot be a criterion to select one of them. Thus additional criteria are required for such cases.

In general (not at extreme prices), in case of volume indeterminacy, the criterion is to maximize traded volume and to share volumes of possible further indeterminacies equally amongst all the markets. Section 10.4 describes how to achieve this goal.

However, because of volume indeterminacy at extreme prices, hierarchical rules have been implemented:

[10] These rules are independent of the ATC or FB network model

- Firstly: avoid curtailment by temporarily matching price taking order locally (see 10.2)
- Secondly: share curtailment (see 10.3).In order to provide fairness, the algorithm will try to equally share the "curtailed volumes" between all markets.
- Thirdly: maximize traded volume (see 10.4)

10.2. Avoiding curtailment

Avoiding curtailment is desirable in general, as price taking orders allow to close positions prior to day-ahead nominations. This is done by be the *local matching constraint* which enforces the local matching of price taking orders prior to the welfare maximization procedure (see Figure 5).

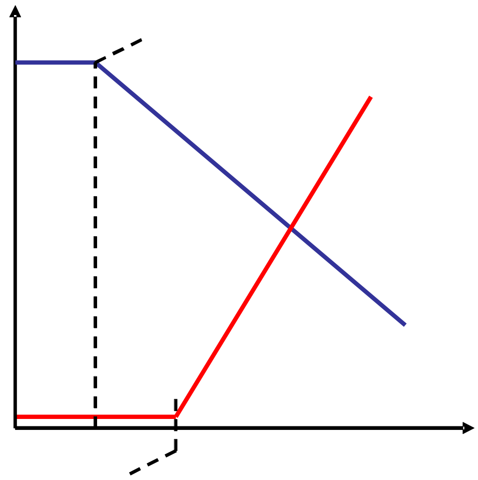


Figure 5 Removing price taking volume from Demand and Supply curves

However, it is possible that a market is initially in curtailment, that is, it is a priori impossible to match locally all price taking orders. In this case only the price taking order that can be met locally can be discarded from the curves, but the remaining price taking volume should be submitted, since not providing this volume could in extreme cases lead to infeasible constraints (e.g. if all markets are initially in curtailment and none would submit any price taking volume then all impose a constraint that they should export. Since no market is willing to import, the balance constraint will be violated). The initial curtailment case is illustrated in Figure 6.

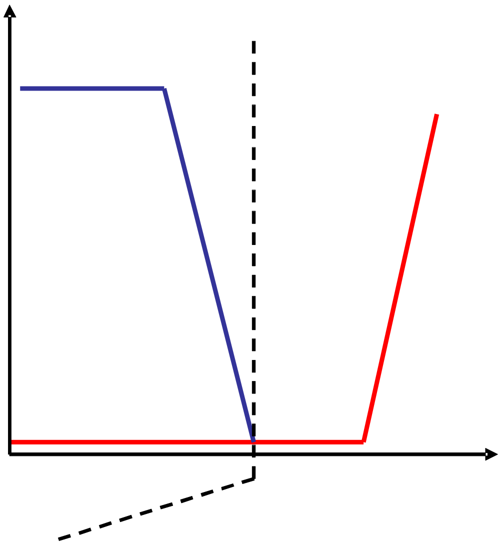


Figure 6 Removing price taking volume from Demand and Supply from a market initially in curtailment

In practice, COSMOS will force the acceptance of price taking hourly supply (respectively demand) orders by matching them with hourly demand (resp. supply) orders. Let us call such a constraint "LocalMatching$_{pmin}$" (resp. "LocalMatching$_{pmax}$").

The local matching constraints can be written as follows:

- For price taking supply orders:

$$LocalMatching_{p\min} : ACCEPT_{Supply_{p\min}} \geq \min\left\{1; \frac{\sum_{d \in Demand} |q_d|}{\sum_{s \in Supply_{p\min}} |q_s|}\right\}$$

- For price taking demand orders:

$$LocalMatching_{p\max} : ACCEPT_{Demand_{p\max}} \geq \min\left\{1; \frac{\sum_{s \in Supply} |q_s|}{\sum_{d \in Demand_{p\max}} |q_d|}\right\}$$

Where

- Supply$_{pmin}$ is the set of all supply orders at minimum price (i.e. price taking),
- Demand$_{pmax}$ is the set of all demand orders at maximum price (i.e. price taking).

The local matching constraints are enforced for each market individually prior to the welfare maximization process. This rule helps finding block selections where curtailment is reduced. Upon lifting volume indeterminacies, the local matching constraint is dropped in order to allow a fair sharing of curtailment.

10.3. Minimizing and sharing curtailment

When one or more markets are initially curtailed (see above), the curtailment might not be avoided through local matching. In this case, the secondary goal is to minimize the volume curtailed. However two additional issues must be dealt with:

1. The solution must be uniquely defined

2. The solution must be fair in apportioning the curtailment between two (or more) markets initially in curtailment.

To this end, the objective is defined as a quadratic function of the acceptance of the curtailed orders:

$$\min \sum_{o \in C} q_o \left(1 - ACCEPT_o\right)^2$$

Subject to the following constraints:

- All market and network constraints
- B_ACCEPT_o = 1 if the block order o is executed in the feasible solution
  = 0 otherwise

where C is the set of price taking orders of markets of whom the curtailment has to be apportioned and q_o is the volume of these price taking orders. In plain English, we minimize the sum of the non-acceptance of price taking orders, measuring non-acceptance by the square of the rejected proportion of the price taking order times its volume. This objective function has several desirable properties:

- it is strictly convex in the space of price taking orders of markets in curtailment, thus guaranteeing uniqueness of solution for these orders
- It will apportion curtailment according to the size of initial curtailment. For example suppose two markets initially in curtailment by X and Y MW are coupled (with infinite capacities) with a third market that can offer Z MW before being itself in curtailment. Then X/(X+Y)*Z will be given to reduce curtailment in market X and Y/(X+Y)*Z will be used to reduce curtailment in market Y. Therefore, each market will see its curtailment decrease by the same proportion Z/(X+Y). Thus price taking orders in each curtailed market are treated equally.

If two markets are curtailed to a different degree, the market with the least severe curtailment (in proportion) would help the other reducing its curtailment, so that both markets end up with identical curtailment ratios.

10.4. Maximizing traded volume

We assume now that for market at P_{min} and P_{max} the volume indeterminacy has been settled according to previous section, and volume in these markets in curtailment is fixed. We turn now our attention to dealing with volume indeterminacy for markets and/or hours not in curtailment. Here our goal will be to maximize traded volume.

We therefore solve the following optimization problem:

$$\min \sum_{o \in H} q_o \left(1 - ACCEPT_o\right)^2$$

Subject to the following constraints:

- All market and network constraints
- B_ACCEPT_o = 1 if the block order o is executed in the feasible solution
  = 0 otherwise

where H is the set of supply and demand step curves that are exactly at the market clearing price of their respective markets and are not at Pmin or Pmax. Again, we want to minimize the rejection of all (demand & supply) orders and apportion this equally across markets.

10.5. Summary

COSMOS lifts potential volume indeterminacies with the following priorities:

1. **Avoiding curtailment of price taking orders whenever possible**
2. **Minimizing and sharing the curtailment when unavoidable**
3. **Maximizing the volume traded in other cases.**